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AN INTEGRAL TREATMENT OF COMBINED BODY FORCE AND FORCED CONVECTION IN LAMINAR FILM CONDENSATION†

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Abstract—Solution of the problem of laminar film condensation with both a gravitational type body force and a moving vapor cocurrent and parallel to the surface is presented in this paper. The solution assumes that the inertia terms of the liquid film have negligible effect, and that the temperature distribution across the film is linear. Results of the solution are presented for the case of Freon 113 and compared with experimental results. It is demonstrated, where both the body force and vapor velocity are significant, that errors of up to 17 per cent in the heat transfer can easily result from neglecting one or the other of these terms.

NOMENCLATURE

a_1, b_1, c_1, etc., constants;
C_p, specific heat;
Δ^*, thickness of vapor boundary layer;
δ^*, liquid film thickness;
δ, Δ, $\frac{\delta^*}{X}(Re_x/Fr_x)^{\frac{1}{2}}, \frac{\Delta^*}{X}(Re_x/Fr_x)^{\frac{1}{2}}$;
Fr_x, $= 1/Z$;
g, gravitational acceleration;
H, defined by equation (11);
h_{fg}, heat of vaporization;
K, thermal conductivity;
μ, ρ_L/ρ_V;
ν, kinematic viscosity;
Nu, Nusselt number;
Pr, Prandtl number;
ρ, density;
R, defined by equation (10);
Re, $= UX/\nu$;
Re_{Lx}, $= U_\infty X/\nu_L$;
T, temperature;
τ, shear stress;
U, velocity in X-direction;
U_∞, free stream vapor velocity;
U_1, $= U/U_\infty$;
V, velocity in Y-direction;
V_1, $= \frac{V}{U_\infty} Fr_x^{\frac{1}{2}}$;
X, coordinate along plate surface;
Y, distance from plate surface;
Y_1, $= \frac{Y}{X}\left(\frac{Re_x}{Fr_x}\right)^{\frac{1}{2}}$;
Z, nondimensional distance, $\frac{gX}{U_\infty^2}$;
ζ, defined by equation (17).

Subscripts
L, liquid;
V, vapor;
W, wall;
x, at position X;
SAT, at saturation conditions;
δ, at the edge of the liquid layer;
∞, at free stream conditions.

1. INTRODUCTION

THE PROBLEM of film condensation heat transfer has been one of classical interest to investigators in the field of heat transfer. Ever since the development by Nusselt [1] in 1916 for condensation on a vertical flat plate, there has been sporadic publications on the problem. Until

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1956 these works have been of an empirical nature based on the needs for improving condenser designs. In 1956, Rohsenow [2] using an integral solution determined the effect of the inclusion of heat capacity in the liquid film.

In 1959, Sparrow and Gregg [3] investigated the effects of the liquid inertia terms by treating the problem within the framework of boundary-layer mathematical techniques. However, they neglected the effects of interfacial shear between the vapor and liquid. They showed little effect for inclusion of the inertia terms for liquids with Prandtl number greater than ten.

In 1961, several authors [4, 5] using boundary-layer similarity techniques solved the laminar body-force-only film-condensation problem, including the interfacial shear as well as the inertia terms.

In the area of forced convection laminar film condensation, there are three analytical papers of importance. These are the papers by Rohsenow, Webber and Ling [6]; by Cess [7]; and by Koh [8]. References [7] and [8] are boundary-layer-type similarity solutions. Reference [7] solves the asymptotes to the problem without a body force, and [8] solves the same problem exactly. Reference [6] neglects the inertia terms and heat capacity of the film in solving the problem of body force with a constant interfacial shear between the liquid and vapor. The results of [6] show reasonable results for condensation in long tubes.

Although Rohsenow *et al.* [6] included a body force in their analysis, the only real attempt to solve the problem of combined body force with forced convection prior to the present analysis was made by Chung [9]. Chung solved the forced convection case by a Blasius-type similarity solution, assuming that the interfacial velocity was zero in determining the vapor boundary-layer thickness and different from zero for the liquid layer equations. He then used a perturbation of this type of solution to solve the case of large Froude number. By this it is meant that the body force is small compared to the interfacial shear force.

In the present investigation, the complete range of Froude number from zero to infinity is covered. This automatically eliminates perturbation solutions and, since there is no similarity solution [7], the problem is approached using the Pohlhausen integral technique. Analytical results are presented for Freon 113 and compared with the experimental results of [10].

2. BASIC EQUATIONS

As the theoretical model for this problem of combined body force with forced convection laminar film condensation, the flat plate shown in Fig. 1 is assumed. The flat plate is suspended

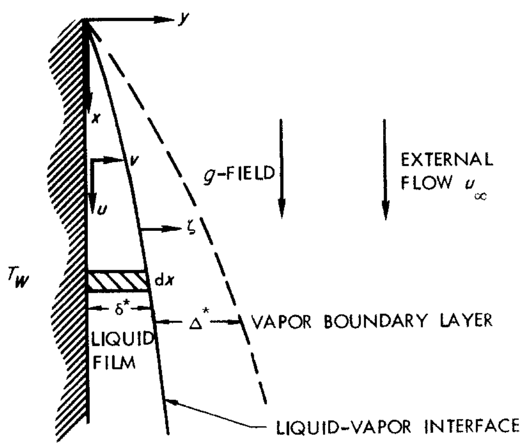

FIG. 1. Theoretical model.

in a flow of saturated vapor with a gravitational field acting in the same direction as the flow which is parallel to the plate. The plate is maintained at a constant temperature, T_W, where $T_W < T_{SAT}$. It is assumed that steady state exists; that is, there are constant liquid and vapor layers with no wave motion on the liquid layer. It is further assumed that the difference between T_W and T_{SAT} is such that the properties of the liquid may be assumed constant. However, the difference between the liquid vapor densities significantly affect the size of the body force. In general, though, $\rho_L \gg \rho_V$.

Two additional assumptions are made. It is assumed that the inertia terms in the liquid have negligible effect and that the temperature profile is linear. From references [3–5, 7, 8] it is seen that, for one driving force and Prandtl numbers of order-of-magnitude greater than one, the inertia terms are indeed negligible. There appears to be no reason why the combination of body force and forced convection should change this. From reference [2], the effect of the heat capacity of the film is negligible if $(1 + 0{\cdot}34C_P(T_{SAT} - T_W)/h_{fg})^{\frac{1}{4}}$ is approximately equal to one. If $C_P/T_{SAT} - T_W)/h_{fg}$ is one or less, the heat transfer is changed by less than 7 per cent, and where it is less than one-tenth, the heat transfer is changed by less than one per cent. As an example, for steam, the effect of assuming a linear temperature profile would be less than one per cent for $(T_{SAT} - T_W)$ from zero to 100 degF.

Under the assumptions listed above, the liquid film can be described by the continuity equation, force balance and the energy balance across the film, which are listed below:

$$\frac{\partial U_L}{\partial X} + \frac{\partial V_L}{\partial Y} = 0 \tag{1}$$

$$0 = g\frac{\rho_L - \rho_V}{\rho_L} + \nu_L\frac{\partial^2 U_L}{\partial Y^2} \tag{2}$$

$$\frac{k_L(T_{SAT} - T_W)}{\delta^*\rho_L h_{fg}} = \frac{d}{dX}\int_0^{\delta^*} U_L\, dY \tag{3}$$

Since the vapor above the liquid film is at its saturation temperature, it is merely necessary to write the momentum and mass-conservation equations to describe it.

$$U_V\frac{\partial U_V}{\partial X} + V_V\frac{\partial U_V}{\partial Y} = \nu_V\frac{\partial^2 U_V}{\partial Y^2} \tag{4}$$

$$\frac{\partial U_V}{\partial X} + \frac{\partial V_V}{\partial Y} = 0 \tag{5}$$

Utilizing the coordinate transformations

$$Z = \frac{gX}{U_\infty^2} = \frac{1}{Fr_x}, \qquad U_1 = \frac{U}{U_\infty},$$

$$Y_1 = \frac{Y}{X}\left(\frac{Re_{Lx}}{Fr_x}\right)^{\frac{1}{2}}, \qquad V_1 = \frac{V}{U_\infty}Fr_x^{\frac{1}{2}}$$

and assuming

$$\frac{\rho_L - \rho_V}{\rho_L} \sim 1{\cdot}0$$

Equations (2)–(5), respectively, become

$$0 = 1 + \frac{\partial^2 U_{1L}}{\partial Y_1^2} \tag{6}$$

$$\frac{C_{P_L}(T_{SAT} - T_W)}{Pr_L h_{fg}} = \delta\frac{d}{dZ}\int_0^{\delta} U_{1L}\, dY_1 \tag{7}$$

$$U_{1V}\frac{\partial U_{1V}}{\partial Z} + V_{1V}\frac{\partial U_{1V}}{\partial Y_1} = \frac{\nu_V}{\nu_L}\frac{\partial^2 U_{1V}}{\partial Y_1^2} \tag{8}$$

$$\frac{\partial U_{1V}}{\partial Z} + \frac{\partial V_{1V}}{\partial Y_1} = 0 \tag{9}$$

where

$$\delta = \frac{\delta^*}{X}\left(\frac{Re_x}{Fr_x}\right)^{\frac{1}{2}}$$

In equations (7) and (8) there still appear two groupings of physical properties, although, they are in nondimensional form. These two groupings will be represented as constants:

$$R = \nu_V/\nu_L \tag{10}$$

$$H = C_{P_L}(T_{SAT} - T_W)/Pr_L h_{fg} \tag{11}$$

The boundary conditions that must be satisfied by equations describing this problem are:

$$X = 0, \quad Z = 0, \quad \delta = 0$$

$$Y = 0, \quad Y_1 = 0, \quad U_{1L} = 0, \quad V_1 = 0$$

$$Y = \infty, \quad Y_1 = \infty, \quad U_1 = 0, \quad V_1 = 0$$

In addition, the following compatibility relations must be satisfied at $Y_1 = \delta$

$$U_{1L}|_\delta = U_{1V}|_\delta = U_{1\delta} \tag{12}$$

$$\tau_L|_\delta = \tau_V|_\delta \tag{13}$$

If equation (6) is integrated with respect to Y_1 there results

$$U_{1L} = -\frac{Y_1^2}{2} + C_1(X)Y_2 + C_2(X) \tag{14}$$

Application of the boundary condition at $Y_1 = 0$ and the compatibility equation (12) yields

$$U_{1L} = \left(\frac{U_{1\delta}}{\delta} + \frac{\delta}{2}\right) Y_1 - \frac{Y_1^2}{2} \tag{15}$$

which describes the velocity profile in the liquid film as a function of the two functions of X, $U_{1\delta}$ and δ.

To determine the function $U_{1\delta}$ and δ requires knowledge of the shear at the interface and use of the energy balance given by equation (7). The momentum equation in the vapor layer must be solved together with the shear compatibility (13) to determine the interfacial shear.

If the edge of the viscous layer in the vapor is defined to be at $Y_1 = \delta + \Delta$, the vapor momentum equation (8) can be integrated with respect to Y_1 from $Y_1 = \delta$ to $Y_1 = \delta + \Delta$ to yield with the aid of equation (9)

$$\frac{\mathrm{d}}{\mathrm{d}Z} \int_{\delta}^{\delta+\Delta} U_1^2 \, \mathrm{d}Y_1 - \frac{\mathrm{d}\delta}{\mathrm{d}Z} - \frac{\mathrm{d}\Delta}{\mathrm{d}Z} + \frac{\mu H U_{1\delta}}{\delta} = R \left.\frac{\partial U_{1V}}{\partial Y_1}\right|_{\delta}^{\delta+\Delta} \tag{16}$$

where $\mu = \rho_L/\rho_V$.

In order to evaluate the vapor momentum equation, it is next necessary to develop a velocity profile for the vapor boundary layer. The conditions that must be satisfied by the vapor velocity profile are:

$$Y_1 = \delta \qquad U_{1V} = U_{1\delta_V} = U_{1\delta_L}$$

$$Y_1 = \delta + \Delta, U_{1V} = 1, \frac{\partial U_{1V}}{\partial Y_1} = 0$$

Proceeding with the development of the vapor velocity profile, first define

$$\zeta = Y_1 - \delta \tag{17}$$

Then, since there are three conditions to be satisfied, the velocity profile can be represented by

$$U_{1V} = a + \frac{b\zeta}{\Delta} + c\frac{\zeta^2}{\Delta^2} \tag{18}$$

Application of the boundary conditions

$$Y_1 = \delta \qquad \zeta = 0 \quad U_{1V} = U_{1\delta}$$

$$Y_1 = \delta + \Delta \quad \zeta = \Delta \quad U_{1V} = 1 \qquad \frac{\partial U_{1V}}{\partial \zeta} = 0$$

yields

$$U_{1V} = U_{1\delta} + (1 - U_{1\delta})\left(\frac{2\zeta}{\Delta} - \frac{\zeta^2}{\Delta^2}\right) \tag{19}$$

Before substituting (19) into the vapor momentum equation, Y_1 must be transformed to ζ. Equation (16) becomes

$$\frac{\mathrm{d}}{\mathrm{d}Z} \int_0^{\Delta} U_{1V}^2 \, \mathrm{d}\zeta - \frac{\mathrm{d}\delta}{\mathrm{d}Z} - \frac{\mathrm{d}\Delta}{\mathrm{d}Z} + \frac{\mu H U_{1\delta}}{\delta} = R \left.\frac{\partial U_{1V}}{\partial \zeta}\right|_0^{\Delta} \tag{20}$$

Substituting (19) into (20) yields

$$\left[\frac{3U_{1\delta}^2}{15} + \frac{4}{15}U_{1\delta} - \frac{7}{15}\right]\delta\frac{\mathrm{d}\Delta}{\mathrm{d}Z} + \left(\frac{2}{5}U_{1\delta} + \frac{4}{15}\right)\delta\frac{\mathrm{d}U_{1\delta}}{\mathrm{d}Z} - \frac{1}{2}\frac{\mathrm{d}\delta^2}{\mathrm{d}Z} + \mu H U_{1\delta} + \frac{2R\delta}{\Delta}(1 - U_{1\delta}) = 0 \tag{21}$$

Equation (13) with the assumption of laminar flow and application of the transformations used previously together with the use of the dimensionless parameters μ and R becomes

$$\left.\frac{\partial U_1}{\partial Y_1}\right|_{\delta} = \frac{R}{\mu}\left.\frac{\partial U_{1V}}{\partial Y_1}\right|_{\delta} \tag{22}$$

Application of the velocity profiles defined in equations (15) and (19) to (22) yields

$$\frac{U_{1\delta}}{\delta} - \frac{\delta}{2} = \frac{2R}{\mu}\frac{(1 - U_{1\delta})}{\Delta} \tag{23}$$

Equation (23) may be solved for

$$\Delta = \frac{2R}{\mu}\frac{(1 - U_{1\delta})}{U_{1\delta} - \delta^2/2} \tag{24}$$

Thus equation (24) defines the vapor boundary-layer thickness in terms of U_δ and δ.

Substitution of (24) into (21) yields

$$\left[\frac{R}{\mu}\left(\frac{U_{1\delta}^2}{5}+\frac{4}{15}U_{1\delta}-\frac{7}{15}\right)\frac{(U_{1\delta}+\delta^2/2)}{(U_{1\delta}-\delta^2/2)^2}\right.$$
$$\left.\times(1-U_{1\delta})-\tfrac{1}{2}\right]\frac{\mathrm{d}\delta^2}{\mathrm{d}Z}$$
$$+\left[\left(\frac{2}{5}U_{1\delta}+\frac{4}{15}\right)\frac{2R}{\mu}\frac{(1-U_{1\delta})\delta^2}{(U_{1\delta}-\delta^2/2)}\right.$$
$$-\frac{2R}{\mu}\frac{(U_{1\delta}^2/5+\frac{4}{15}U_{1\delta}-\frac{7}{15})}{(U_{1\delta}-\delta^2/2)^2}$$
$$\left.\times(1-\delta^2)\delta^2\right]\frac{\mathrm{d}U_{1\delta}}{\mathrm{d}Z}$$
$$+\mu[(H+1)U_{1\delta}-\delta^2/2]=0 \qquad (25)$$

Equation (7) may be written in terms of the liquid velocity profile.

$$H=\delta\frac{\mathrm{d}}{\mathrm{d}Z}\int\limits_0^\delta\left[\frac{U_{1\delta}Y_1}{\delta}\right.$$
$$\left.+\frac{\delta^2}{2}(Y_1/\delta-Y_1^2/\delta^2)\;\mathrm{d}Y_1\right] \qquad (26)$$

Carrying out the operations indicated in (26) yields

$$H=\frac{\delta^2}{2}\frac{\mathrm{d}U_{1\delta}}{\mathrm{d}Z}+\left(\frac{U_{1\delta}}{4}+\frac{\delta^2}{8}\right)\frac{\mathrm{d}\delta^2}{\mathrm{d}Z} \qquad (27)$$

Now the problem of combined body force with forced convection has been reduced to the determination of two unknown functions of Z: $U_{1\delta}$ and δ^2. These two functions will result from the combined solution of equations (25) with (27).

The solutions of references [5, 6] indicated that, for body-force-only $U_{1\delta}$ and δ^2, both increase as $X^{\frac{1}{2}}$. For forced convection, reference [9] indicates that $U_{1\delta}$ is a constant and δ^2 varies as X. It would thus appear that $U_{1\delta}$ should be a function of X, (Z), somewhere between a constant and a function of $X^{\frac{1}{2}}$, and that δ should be a function somewhere between $f(X^{\frac{1}{4}})$ and $f(X^{\frac{1}{2}})$. In the perturbation of the forced convection problem reference [9] shows a linear first dependence of the interfacial velocity on Z; however, this was based on the assumed form of his solution. This does not guarantee that a singularity in the first derivative of $U_{1\delta}$ with Z does not exist.

Thus it seems logical that $U_{1\delta}$ and δ^2 could be better represented by infinite series of the forms

$$\delta^2=Z^{\frac{1}{2}}(a_0+a_1Z^{\frac{1}{4}}+a_2Z^{\frac{1}{2}}+a_3Z^{\frac{3}{4}}+\dots) \qquad (28)$$

$$U_{1\delta}=(b_0+b_1Z^{\frac{1}{4}}+b_2Z^{\frac{1}{2}}+b_3Z^{\frac{3}{4}}+\dots) \qquad (29)$$

Substitution of these series into equations (25) and (27) and solving for the coefficients will establish whether singularities exist at $Z = 0$. The nature of equations (25) and (27) do not lend themselves readily to solutions of this nature and therefore are more readily solved by numerical means. However, in order to numerically integrate the equations, it is necessary to determine the first order variation of $U_{1\delta}$ and δ with Z. Substitution of (28) and (29) into (25) and (27) and solving for the first nonconstant terms in the series yields the solution for very small values of Z. Thus, for an asymptotic solution to the forced convection problem there is obtained

$$\delta^2=a_2Z \qquad (30)$$

and

$$U_{1\delta}=b_0+b_4Z \qquad (31)$$

These equations indicate that no singularities exist for δ^2 and $U_{1\delta}$. The coefficients a_2, b_0, and b_4 can be determined from the expressions

$$a_2b_0=4H \qquad (32)$$

$$b_0^3[\mu^2(H+1)-4/5\,HR]+8/15\,HRb_0^2$$
$$+[44/15\,HR-2\mu H]b_0-28/15\,HR$$
$$=0 \qquad (33)$$

and

$$\left[3b_0^2\mu(H + 1) + \frac{38}{15}\frac{R}{\mu}a_2b_0 + \frac{7}{15}\frac{R}{\mu}a_2 - a_2b_0 - \frac{1}{5}\frac{R}{\mu}a_2b_0^2 - \frac{4}{5}\frac{R}{\mu}a_2b_0^3\right]b_4 - a_2b_0^2\left[\mu H + \frac{3}{2}\mu\right] - \frac{7}{30}\frac{R}{\mu}a_2^2 + a_2^2b_0\left(\frac{11}{30}\frac{R}{\mu} + \frac{1}{2}\right) - \frac{Ra_2^2b_0^2}{30\mu} - \frac{1}{5}\frac{R}{\mu}a_2 = 0 \quad (34)$$

Discussion of results

Numerical integration of equations (25) and (27) are reported for one value each for μ and R and for three values of H over the range of Z = 0 to $Z = \infty$. The values of μ and R were selected to compare with the experimental results for Freon 113 of reference [10].

Typical velocity profiles for four values of Z are shown in Fig. 2. These profiles are for Freon 113 at 1 atm and at a $T_{SAT} - T_W = 40$ degF. For this case, at $Z = 5 \times 10^{-4}$, forced convection prevails. At $Z = 6{\cdot}1 \times 10^{-3}$, both body force and forced convection are important. At $Z = 0{\cdot}60$, the body force is dominant with still a small effect due to interfacial shear. At $Z = 40{\cdot}2$, the body force prevails.

Figure 3 shows the dimensionless heat-transfer coefficient $Nu_x Fr_x^{\frac{1}{4}}/Re_x^{\frac{1}{2}}$ as a function of Z, $1/Fr_x$, for Freon 113 with $(T_{SAT} - T_W)$ at values of 10, 40, and 70 degF. These values of $(T_{SAT} - T_W)$ correspond to values of H of 0·00617, 0·02468 and 0·04319, respectively. As with the analyses of other authors [1, 7, 8] for case of only one driving force, the heat-transfer coefficient decreases with increasing $(T_{SAT} - T_W)$ and consequently with increasing values of H. The dashed lines on this figure indicate the asymptotes to this problem. The dashed lines downward to the right are the asymptotes for pure forced convection and have slopes of −0·50 with increasing Z. This indicates that the heat transfer is independent of the Froude number and,

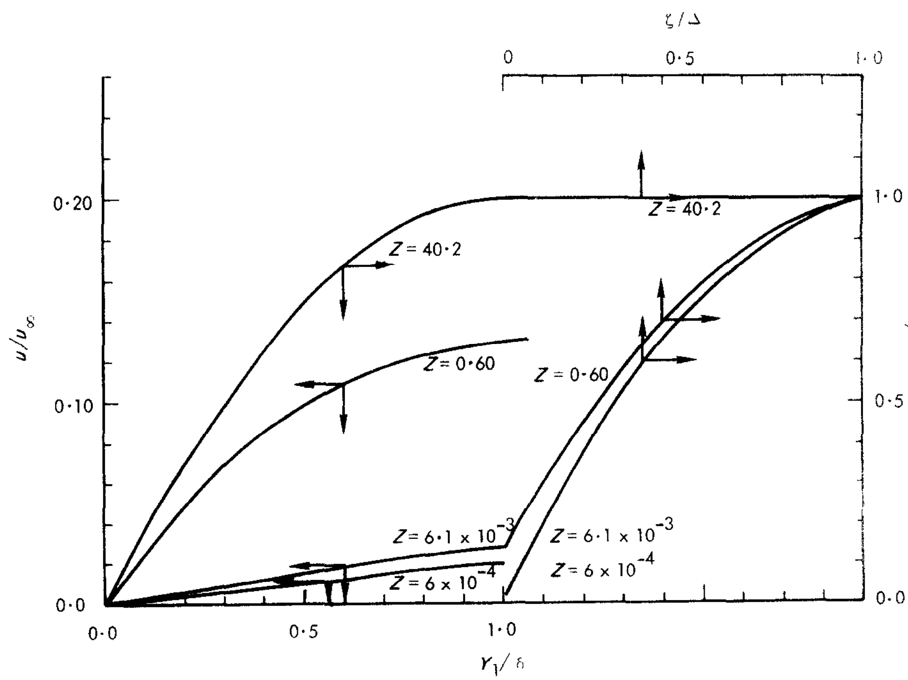

FIG. 2. Velocity profiles for Freon 113 at ΔT_{SAT-W} = 40 degF for several values of Z.

thus, the Grashof number down to a Froude number of about 2000. Conversely, the dashed lines originating at the right are the asymptotes for the case of body force only. The slopes of these lines are $-0{\cdot}25$ in the direction of increasing Z. This indicates that, for Freon 113 for Froude numbers less than 1·0, the problem is nearly one of body force only.

Thus, for $\mu = 210{\cdot}29$, $R = 4{\cdot}495$, both body force and forced convection are important for the range of Froude number between 1 and 2000. Above 2000 the problem can be treated as pure forced convection; below one it can be treated as one where only the body force acts. If one neglected the combined effects of the body force with the forced convection in the region between Froude number equal 1 and 2000, and instead used the asymptote which would yield the larger value, a maximum error of less than 20 per cent would result.

Of some interest is the form of $U_{1\delta}$ as a function of Z, Fig. 4 shows the nondimensional interfacial velocity for the three values of H. The change in velocity is rather gradual until $Z = 3 \times 10^{-3}$. From this point to Z nearly equal to one, the slope of the curve is gradually increasing. From $Z = 1{\cdot}0$ on to higher values the slope of the curve is $+0{\cdot}50$. This is the region where the body force prevails, and it follows the form of Koh [5] for the case of body force only.

Comparison with other analytical solutions

The comparison of the analytical results of this investigation with those of other authors can be made not only for the area of combined body force and forced convection (where little analysis has been made previously), but also for the asymptotes found in this investigation for the cases of body force only and forced convection only. The first comparisons will be made for the asymptotes.

For the case of pure forced convection, the asymptote yields a value of $Nu_x Re_x^{-\frac{1}{2}} = 0{\cdot}465$ for $H = 0{\cdot}02468$, $\mu = 210{\cdot}29$ and $R = 4{\cdot}495$. Koh [8] gives $Nu_x Re_x^{-\frac{1}{2}}$ equal to approximately 0·490 while Cess [7] gives $Nu_x Re_x^{-\frac{1}{2}} = 0{\cdot}50$. This is a deviation of under 5 per cent from Koh's solution [8], which is a solution of the forced convection problem by means of a similarity solution. The deviation from Cess' solution [7] is a slightly higher 7 per cent. However, Cess' solution is only an approximate solution based

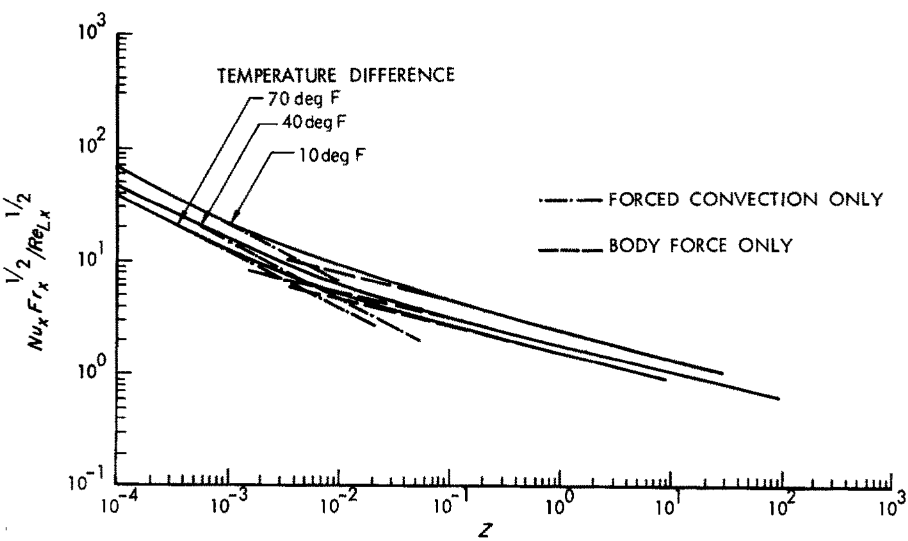

FIG. 3. $Nu_x Fr_x^{\frac{1}{2}}/Re_{Lx}^{\frac{1}{2}}$ vs. Z for Freon 113.

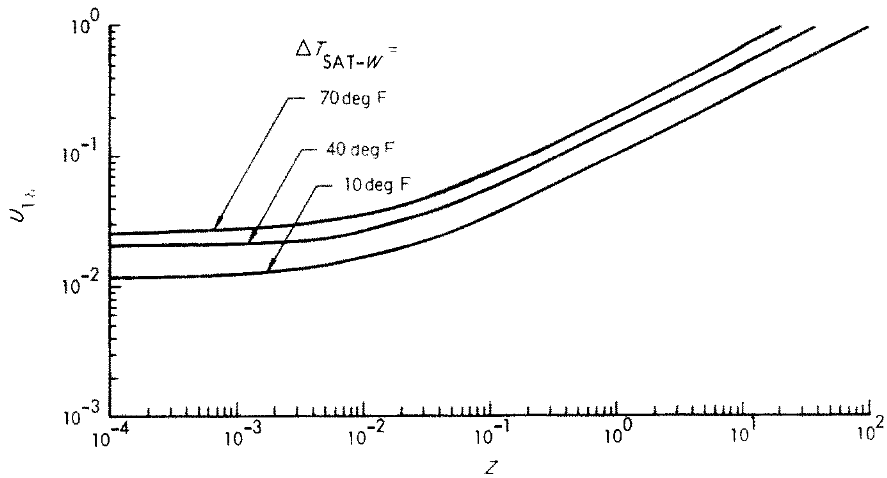

FIG. 4. $U_{1\delta}$ vs. Z for Freon 113 with ΔT_{SAT-W} = 10, 40 and 70 degF

on the similarity equations of Koh [8]. For the other values of H calculated, the same agreement is found with Koh's analysis.

For the case of body force only, the values of local Nusselt number divided by the Nusselt number which can be determined from the analysis of Nusselt [1] are plotted versus H on Fig. 5. Comparison is made with the solutions of [3, 5, 11]. In the range of H investigated in the present analysis, all of the analyses [3, 5, 11] reduce to Nusselt's solution. The solution of Sparrow and Gregg [3] gives a solution 4 per cent above Nusselt's solution at higher values of H. The difference between Sparrow and Gregg's solution and Nusselt's was in the fact that the former included the inertia terms. References [5] and [11] show exact agreement with Nusselt up to a value of H of 7×10^{-2}, and then they show a decrease in the heat transfer for small Prandtl number fluids and for fluids with a Prandtl number of one. The three points determined for Freon 113 by the present analysis indicate that the Nusselt number is the same as predicted by Nusselt [1]. This would be expected since when $U_{1\delta} = \partial U_{1L}/\partial Y_1|_\delta = 0$, equation (14) simplifies to

$$U_1 = \delta Y_1 - (Y_1^2/2) \tag{35}$$

which is exactly the velocity profile determined by Nusselt [1].

Figure 6 shows the comparison of the local nondimensional heat-transfer coefficients for combined body force and forced convection as a function of Z, with the nondimensional heat-transfer coefficients calculated by the method of Rohsenow *et al.* [6] and by Chung [9].

The comparison is shown for H = 0·02468, μ = 210·20, and R = 4·495. From Chung [9] there is only *one point*, the place where the heat transfer deviates by 10 per cent from the forced convection asymptote. This point falls exactly on the curve determined in the present analysis. The solution of Rohsenow *et al.* [6] gives the same value as obtained in this analysis for the local nondimensional heat transfer at the end of the nondimensional length over which the vapor shear was assumed constant. However, up to that point it underestimates the nondimensional heat-transfer coefficient. If, for example, the plate had a nondimensional length Z_L = 1·0 then, over the first one-tenth of the length of the plate, the nondimensional heat-transfer coefficient would be underestimated and, over the remaining nine-tenths of the length values, approximately the same as in the present analysis would be obtained. However, for plates with a Z_L, less

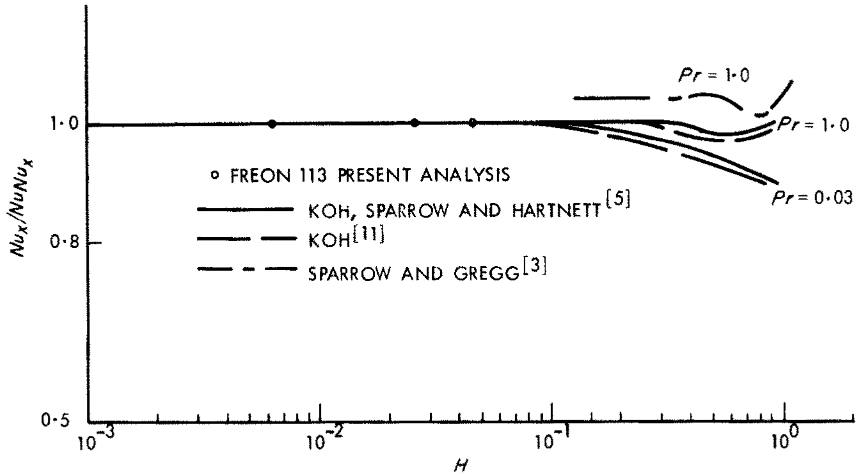

FIG. 5. Comparison of body-force-only asymptote with solutions of other investigators.

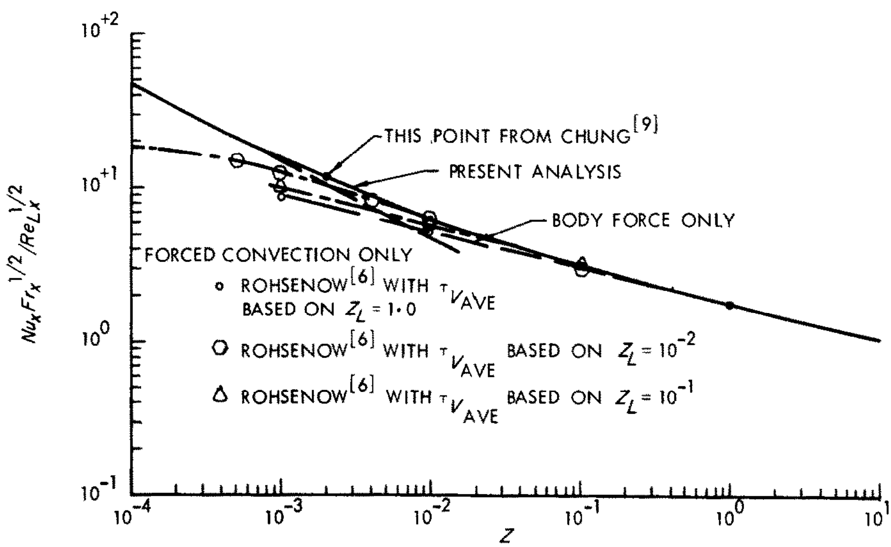

FIG. 6. Comparison of analytical results for combined body force with forced convection with that of other investigators $M = 210{\cdot}29$, $R = 4{\cdot}495$, $H = 0{\cdot}02468$.

than 1·0 the deviation becomes worse. It appears, though, that if one used the technique of Rohsenow [6] but altered it such that one divided the plate into increments and assumed τ_V constant over each of these intervals, but of different magnitudes, one would achieve a solution in good agreement with this analysis. This can be seen by observing the end points of the three curves shown on Fig. 6 based on Rohsenow's [6] method but with different values of $\tau_{V_{ave}}$.

Comparison between theoretical and experimental results

In Fig. 7, comparison is made between the theory for pure forced convection and the experimental results for Freon 113 of reference [10].

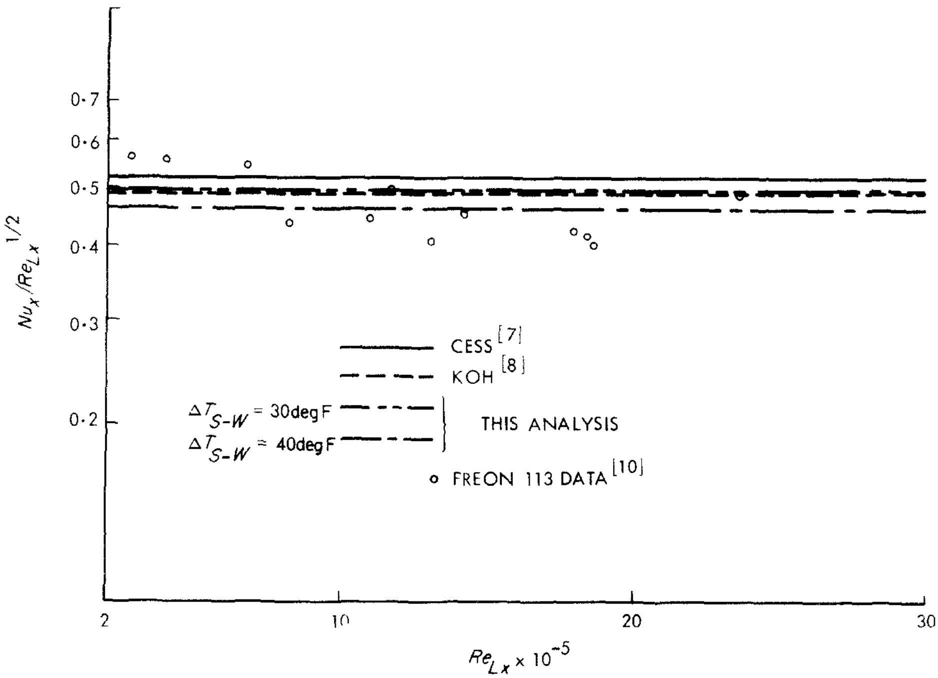

FIG. 7. Comparison of theoretical and experimental results for pure forced convection.

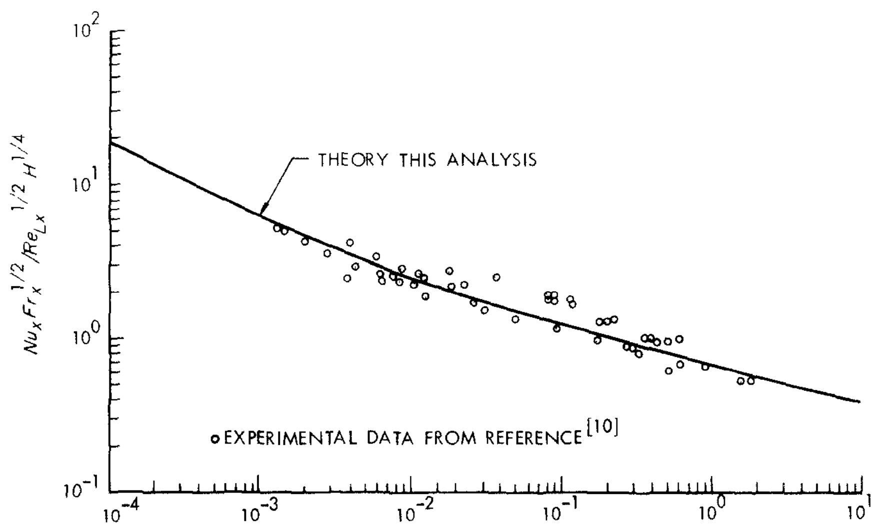

FIG. 8. Comparison of theoretical and experimental results for combined body force with force convection from Freon 113.

The present theory shows as good agreement with the experimental data as does Koh [8] or Cess [7]. Reference [10] indicated that much of the data scatter was probably due to the presence of air in the vapor. However, the theory does show agreement within ±20 per cent.

In Fig. 8, the data of reference [10] for combined body force with forced convection has been plotted. The ordinate of this graph is $Nu_x Fr_x^{\frac{1}{2}} Re_x^{-\frac{1}{2}} H^{\frac{1}{4}}$. This was chosen since the values of $(T_{SAT} - T_W)$ for the experimental data varied. Reference [10] showed that the factor of $H^{\frac{1}{4}}$ could be used to remove the dependence of $(T_{SAT} - T_W)$ of the experimental data for both body-force-only and forced-convection-only over the range of $(T_{SAT} - T_W)$ encountered in the experiment. However, it should be emphasized that this type of correlation for the forced-convection-only data would not hold over a very wide range of $(T_{SAT} - T_W)$. For the combined body force with forced-convection theoretical solutions obtained for Freon 113 with $(T_{SAT} - T_W)$ of 10 and 40 degF (which encompasses all experimental data), it is found that multiplying them by $H^{\frac{1}{4}}$ also reduced them to a single line and thus validates the choice of the ordinate. Again it is emphasized that the variation of the dimensionless heat-transfer coefficient with $H^{\frac{1}{4}}$ *is not a conceptual fact*, but it is a matter of circumstance for the limited range of H investigated.

3. CONCLUSIONS

The mathematical results for the body-force-only asymptote agrees favorably with all other authors [1–5]. Good agreement is also shown between the forced convection asymptote and the results of Cess [7] and Koh [8].

For combined body force with forced convection, the present analysis agreed with the perturbation solution of Chung [9] for the case of large Froude numbers. Thus, the solution reported appears to define the problem properly and give correct solutions within the limitations imposed by the assumptions made in the analysis.

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Résumé—On expose la solution du problème de la condensation par film laminaire avec effets de la gravité et d'un écoulement de vapeur parallèle à la surface et dans le même sens que l'écoulement dans le film. On suppose que les termes d'inertie dans le film liquide ont un effet négligeable, et que la distribution de température à travers le film est linéaire. Les résultats théoriques sont donnés dans le cas du Fréon 113 et sont comparés avec les résultats expérimentaux. On démontre que, lorsque la force volumique et la vitesse de la vapeur sont importantes, des erreurs sur le transport de chaleur allant jusqu'à 17 pour cent peuvent facilement se produire lorsqu'on néglige l'un ou l'autre de ces termes.

Zusammenfassung—Die Lösung des Problems der laminaren Filmkondensation unter Berücksichtigung sowohl der Schwerkraft als auch eines, zur Oberfläche parallelen, gleichgerichteten Dampfstroms wird hier mitgeteilt. Für die Lösung ist angenommen, dass der Einfluss der Trägheitskräfte aur den Flüssigkeitsfilm vernachlässigbar ist und die Temperatur im Film linear verläuft. Ergebnisse der Lösung werden für Freon 113 angegeben und mit experimentellen Resultaten verglichen. Es wird gezeigt, dass dort, wo sowohl die Massenkraft als auch die Dampfgeschwindigkeit von Einfluss sind, bei Vernachlässigung der einen oder anderen, in der Berechnung des Wärmeubergangs Fehler bis 17 prozent auftreten können.

Аннотация—В статье приведено решение задачи ламинарной пленочной конденсации при одновременном воздействии гравитационной массовой силы спутного и параллельного к поверхности течения пара. Предполагается, что инерционные члены для жидкой пленки играют незначительную роль и что температура распределяется линейно поперек пленки. Результаты решения представлены для фреона 113. Дано сравнение с экспериментальными результатами. Показано, что при расчете коэффициента теплообмена, когда как массовая сила, так и скорость пара играют важную роль, пренебрежение одним из этих членов может привести к ошибке до 17%.